Notes 12/14 - All about i

Definition

$$i = \sqrt{-1} \quad \text{OR} \quad i^2 = -1$$

- We now can take the $\sqrt{}$ of a negative, but our answer is imaginary

Square roots of negative numbers

→ need i to $\sqrt{}$ a negative!

Ex 1: $\sqrt{-9} = \sqrt{9}\sqrt{-1} = 3i$

(since $-9 = 9 \cdot (-1)$; simplified)

Ex 2: $\sqrt{-25} = \sqrt{25}\sqrt{-1} = 5i$

Ex 3: $\sqrt{-4x^2} = \sqrt{4}\sqrt{-1}\sqrt{x^2} = 2xi$

Do: 1) $\sqrt{-49}$

2) $\sqrt{-144}$

3) $\sqrt{-36y^2}$

Adding and Subtracting Complex Numbers

A complex number : $a + bi$

a: real component (no i) ↑ bi: imaginary component (i) ↑

Ex: $3+2i$, $8-7i$

Just treat i like a variable when $+, -$ complex #s
Combine Like terms!

Ex 1: $(3+2i) + (5+6i) = \boxed{8 + 8i}$

$3+5$ ↑ $2i+6i$

Ex 2: $(5+3i) - (2-4i)$

$5+3i \quad -2+4i = \boxed{3+7i}$

Do: $(5+6i) + (9+3i)$

2) $(7+4i) + (3-5i)$

3) $(6-2i) - (4+9i)$

4) $(-2+3i) - (6-8i)$

Multiplying Imaginary Numbers

- Do the first steps like normal
- When you get i^2, simplify $i^2 = -1$

Ex 1: $(6i)(2i) = 12i^2 = 12(-1) = -12$ (since $i^2 = -1$)

Ex 2: $(-4i)(3i) = -12i^2 = -12(-1) = 12$

Ex 3: $f(x) = 2x^2 + 1$, find $f(3i)$

$$f(3i) = 2(3i)^2 + 1$$
$$= 2(9i^2) + 1$$
$$= 18i^2 + 1$$
$$= 18(-1) + 1 = -18 + 1 = \boxed{-17}$$

Do:

1) $(-4i)(5i)$

2) $(10i)(3i)$

3) $f(x) = 3x^2 - 4$, find $f(2i)$

Multiplying Complex #5

- Do all of the first steps like normal!
- At the end, make $i^2 = -1$ and keep simplifying

Ex 1: $(2+4i)(3-i)$

$= 6 - 2i + 12i - 4i^2$

$= 6 + 10i - 4i^2$

$= 6 + 10i - 4(-1)$ ← since $i^2 = -1$

$= 6 + 10i + 4$

$= \boxed{10 + 10i}$

Ex 2: $(-4+5i)(-4-5i)$

$= 16 + 20i - 20i - 25i^2$

$= 16 - 25i^2$

$= 16 - 25(-1)$

$= 16 + 25$

$= \boxed{41}$

DO: $(6+4i)(9-3i)$

2) $(2-6i)(2+6i)$

Powers of i

* The trick is to find groups of i^2

$i^{\text{even \#}}$ → all i's get paired up evenly (answer is 1 or -1)

$i^{\text{odd \#}}$ → i left out front (don't get paired up evenly) (answer is i or $-i$)

Ex 1: $i^6 = (i^2)^3 = (-1)^3 = -1$

(i^2: 3 sets of i^2; since $i^2 = -1$)

Ex 2: $i^8 = (i^2)^4 = (-1)^4 = 1$

Ex 3: $i^3 = i \cdot i^2 = i(-1) = -i$

Ex 4: $i^{13} = i \cdot i^{12} = i(i^2)^6 = i(-1)^6 = i(1) = i$

Do:

1) i^4

2) i^5

3) i^{30}

4) i^{33}